Influence of viscosity on the diffraction of sound by a periodic array of screens

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The paper contains an analysis of the transmission of a pressure wave through a periodic grating including the influence of the air viscosity. The system of equations in this case consists of the compressible Navier–Stokes equations associated with no-slip boundary conditions on solid surfaces. The problem is reduced to two hypersingular integral equations for determining the velocity components along the slits. These equations are solved by using Galerkin's method with some special trial functions. The results can be applied in designing protective screens for miniature microphones realized in the technology of micro-electro-mechanical systems (MEMS). In this case, the physical dimensions of the device are on the order of the viscous boundary layer so that the viscosity cannot be neglected. The microfluidic model of the screen consists of a periodic array of slits in a substrate. The analysis indicates that the openings in the screen should be on the order of 10 μm in order to avoid excessive attenuation of the signal.
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I. INTRODUCTION

The reflection and transmission of a scalar plane wave through a periodic grating is a classical problem of acoustics. Thus, Lamb[1] succeeded in obtaining analytical formulas for the reflection and transmission coefficients in the low-frequency range for normal incident waves; Miles[2] obtained a one-mode approximation for small screens, in the case of oblique incidence. Achenbach and Li[3] developed a method that is appropriate for arbitrary frequencies and angles of incidence. They used a representation of the solution as an integral over the length of a screen. Finally, we mention the results in Ref. 4, where explicit analytical formulas are given for the reflection and transmission coefficients in one-mode, oblique incidence penetration.

The inclusion of viscous effects in acoustics is a subject not very often approached. The book by Pierce[5] contains a chaper discussing the dissipative processes devoted especially to explain attenuation of sound waves. Davis and Nagem, in Ref. 6, have investigated the problem of diffraction by a half plane studying the behavior of fluid velocity near a diffracting edge. The same authors analyzed, in Ref. 7, diffraction of an acoustic plane wave by a circular aperture in a viscous fluid.

The viscous dissipative processes are described by a constitutive relationship between shear stress and rate of shear involving the shear viscosity μ and the bulk viscosity μ_B. The bulk viscosity takes into account the departure of the kinematic mode of molecular motion from mutual thermodynamic equilibrium. By assuming a Newtonian constitutive relationship, the momentum equations yield the Navier–Stokes system corresponding to a compressible fluid. In the case of isentropic flow and constant viscosity (μ), the system decomposes into an acoustical (propagational) mode and a vorticity (viscous) mode. The vorticity mode dies out rapidly with increasing distances from boundaries, interfaces, and sources. Therefore, in the bulk of the atmosphere, the velocity and the pressure fields are described mainly by the propagating mode.

A simple calculation shows that the viscous mode gives, along a solid boundary, a viscous boundary layer of thickness $t_{\mathrm{visc}}=\sqrt{\mu/(\pi f \rho)}$ (f denotes the frequency and ρ the density)[5] which has the value 223 μm at 100 Hz and 22.3 μm at 10 KHz. In the case of miniature silicon microphones, realized in MEMS technology, the linear dimensions of the device are of the order of 1 mm. Hence, the viscous boundary layer cannot be neglected anymore in determining the disturbance of the sound waves by the microphone parts. As an example, we consider the influence of a protecting system, consisting of a plane surface containing a periodic system of parallel slits (a horizontal periodic grating), placed in front of the diaphragm. This analysis is important since we are interested in protective surfaces having small holes, which will avoid the penetration of water and dust particles to the diaphragm surface. It is clear that for very narrow slits the transmission coefficient is small. Therefore, we have to find an optimum dimension of the holes which enables the penetration of the sound and at the same time does not allow the penetration of water and dust particles.

In Sec. III we give representation formulas for the pressure and velocity fields in the upper and lower half planes. The incoming wave is considered as a pressure wave characterized by the angle θ_0. As the attenuation of the sound waves in air is very small, we neglect it in the incoming plane wave and in the propagating modes. This is why, despite the viscous dissipation, we continue to use the Sommerfeld condition for selecting the proper waves in each case. The representation formulas for the scattered and transmited

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pressures contain an infinite number of wave modes, each with its cutoff frequency. At the cutoff frequency, a mode converts from an evanescent mode into a propagating wave mode. At small frequences only the lower-order modes are propagating. As the frequency is increasing, more and more evanescent modes convert to propagating modes. In the case of acoustical frequences in air, only the lowest mode is propagating. This is the case we are considering in this paper. The case when other modes are also propagating can be analyzed similarly. By using the momentum equations we obtain representation formulas for velocities. These contain, besides the above-discussed modes, some viscous (vorticity) modes which are decaying exponentially with the distance to the perturbation sources.

Next, we consider as main unknown functions the velocity components along the slits. The advantage, as compared with the approach in Ref. 3, where the unknown function is related to the pressure on the screens, is that the final integral equations are simpler. All the coefficients entering into representation formulas can be determined in terms of Fourier coefficients of the velocity components on the slits. Now, the condition of continuity of velocity and its normal derivative along the slits furnishes the functional equations for solving the problem. There is one such equation for each of the velocity components. As these equations contain some divergent Fourier series, they can be interpreted properly only within distribution theory. Further on, we succeeded in transforming the distributional equations into hypersingular integral equations. The singular part of both equations is the same; the weak singular parts differ by a multiplicative constant and the regular parts contain continuous functions resulting from summing some uniform convergent Fourier series.

In Sec. V we developed a method for solving the hypersingular integral equations based on the representation of solutions in terms of a basis of functions, given by some Chebyshev functions, and using also the convenient form of the convolution equations in the spectral domain. With a Galerkin technique we succeeded in obtaining an infinite system of linear equations for each of the integral equations. The systems have good computational properties. Thus, the coefficients of the equations result from using the spectral properties of the singular operators, the FFT transform of some smooth functions (realized in fact by using the 2D discrete cosine transform of MATLAB), and the summation of some fast convergent infinite series. The finite sections of the final systems are well conditioned such that we need only a small number of terms to obtain a solution with good precision.

Section VI contains some numerical results. We computed the transmission coefficient for certain geometries important in designing miniaturized microphones. We applied the same mathematical technique to the classical (nonviscous) acoustical periodic grating problem. The results are also given in Sec. VI for comparison. We note that the numerical results obtained in the nonviscous case coincide with those calculated by Ref. 4. The graphs show that for very small slit width the influence of viscosity is very important.

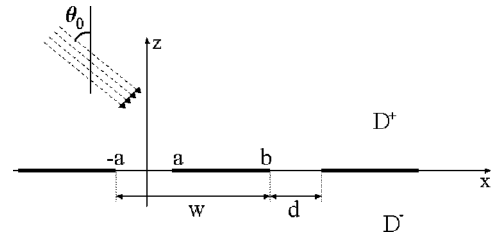

FIG. 1. The geometry of the problem.

The graphs provided can be used in designing of microphones realized in MEMS technology.

Finally, in the Appendix we give the most important formulas used in calculation of the spectral form of singular integral operators and also of the regular parts of the integral equations.

II. THE EQUATIONS OF THE PROBLEM

A. Formulation of the problem

Let us consider the penetration of a pressure wave through the array of coplanar rigid screens located at $z=0$ in Fig. 1. The screens are infinitely long in the y direction. The opening between two neighboring screens is $2a$ and the period of the grating is $T=a+b$. We denote by $\mathcal{D}^+$ the upper half plane $(z>0)$ and by $\mathcal{D}^-$ the half plane $z<0$. The incident wave is located in the domain $\mathcal{D}^+$ and its propagation vector makes an angle $\theta_0-\pi$ with the z axis.

There are two periodic phenomena in this problem: one is associated with the acoustical incoming wave and the other one with the grating periodicity. To avoid possible confusions we associate a "*" with the quantities related to the acoustical incoming wave (k^* is the spatial frequency wave number of the plane incoming wave and ω^* its angular frequency). The "nonstarred" quantities T and $\omega=2\pi/T$ are the spatial period of the grating and its corresponding spatial frequency, respectively.

B. The equations of the motion of a compressible viscous fluid

The isentropic motion of a viscous fluid is described by the continuity equation

$$\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\mathbf{V})=0, \tag{1}$$

and the momentum equation

$$\rho\left(\frac{\partial}{\partial t}+\mathbf{V}\cdot\nabla\right)\mathbf{V}+\nabla\cdot\sigma=\mathbf{0}. \tag{2}$$

Here, by $\mathbf{V}$ we denote velocity, ρ is density, and the stress tensor σ has the components

$$\sigma_{ij}\equiv\sigma_{ij}[P,\mathbf{V}]=\left[P-\left(\mu_B-\frac{2}{3}\mu\right)\nabla\cdot\mathbf{V}\right]\delta_{ij}-\mu\left(\frac{\partial V_i}{\partial x_j}+\frac{\partial V_j}{\partial x_i}\right). \tag{3}$$

Also, μ and μ_B are the shear and bulk viscosities[5] and P the pressure.

Also, in the case of isentropic flow the density is a function of pressure alone such that the state equation can be expressed as

$$\rho=\rho(P). \tag{4}$$

For a viscous fluid we have the nonslip boundary condition

$$\mathbf{V}(\mathbf{x},t)=\mathbf{0}, \tag{5}$$

on any immobile solid surface.

In the case of inviscid (nonviscous) model of fluid, the viscosities μ and μ_B have to be considered zero in (3); also, the boundary condition (5) is replaced by the nonpenetration condition

$$V_n(\mathbf{x},t)=0, \tag{6}$$

stating the cancellation of normal component of velocity on any immobile solid surface.

Besides this, we will impose that all the propagating perturbations, except for the incoming plane wave, are outgoing waves (*Sommerfeld radiation condition*).

C. The equations of the motion of a viscous fluid in linear acoustic approximation

In the case where the coordinate system is chosen so that the unperturbed fluid is at rest, the first-order equations describing the motion of the gas can be written as[8,5]

$$\frac{1}{c_0^2}\frac{\partial}{\partial t}\frac{p'}{\rho_0}+\nabla\cdot\mathbf{v}'=0, \tag{7}$$

$$\frac{\partial\mathbf{v}'}{\partial t}+\nabla\left[\frac{p'}{\rho_0}-(\nu'-\nu)\nabla\cdot\mathbf{v}'\right]-\nu\Delta\mathbf{v}'=\mathbf{0}, \tag{8}$$

where p' and $\mathbf{v}'$ denote the pressure and velocity perturbations, respectively, and

$$\nu=\frac{\mu}{\rho_0},\quad \nu'=\frac{\mu_B}{\rho_0}+\frac{4\mu}{3\rho_0},$$

are the kinematic viscosities.

We consider the case where all the physical variables are harmonic in time with the same angular velocity, $\omega^*=2\pi f$. The case of general time dependence can be obtained, after analyzing each frequency separately, by Fourier superposition. In the case of simple harmonic oscillations in time, we shall write

$$\{p'(\mathbf{x},t),\mathbf{v}'(\mathbf{x},t)\}=\{p(\mathbf{x}),\mathbf{v}(\mathbf{x})\}\exp(-i\omega^* t).$$

In this case the continuity equation (7) becomes

$$\nabla\cdot\mathbf{v}=\frac{i\omega^*}{c_0^2}\frac{p}{\rho_0}. \tag{9}$$

Also, the momentum equation can be written as

$$\Delta\mathbf{v}+\frac{i\omega^*}{\nu}\mathbf{v}=\frac{1+(\nu-\nu')i\omega^*/c_0^2}{\nu}\nabla\frac{p}{\rho_0}. \tag{10}$$

The relationships (9) and (10) give the equation for the pressure

$$[\Delta+k^{*2}]p=0. \tag{11}$$

Here, we have used the notations

$$k^*=\frac{\omega^*}{\sqrt{c_0^2-i\omega^*\nu'}},\quad \mathrm{Im}(k^*)\leqslant 0.$$

Equations (10) and (11) yield the equation for velocity in the form

$$[\Delta+k^{*2}]\left[\Delta+\frac{i\omega^*}{\nu}\right]\mathbf{v}=0. \tag{12}$$

Equation (12) is in fact the product of two operators. Consequently, the solution can be written as a sum of two terms: the first describes a propagation mode (called also the acoustical mode) and the second is a diffusion mode driven by viscosity.

III. THE REPRESENTATION FORMULAS FOR THE PRESSURE AND VELOCITY FIELDS

Let us consider now an incoming pressure plane wave in $\mathcal{D}^+$

$$p^0(x,z)/\rho_0=c_0^2\exp\{ik^*(x\sin\theta_0-z\cos\theta_0)\}. \tag{13}$$

It can be verified directly that (13) satisfies the pressure equation (11) and the corresponding velocity field is $\mathbf{v}^0=u^0(x,z)\hat{\mathbf{x}}+w^0(x,z)\hat{\mathbf{z}}$, where

$$u^0(x,z)=ik^*c_0^2\delta\sin\theta_0\exp\{ik^*(x\sin\theta_0-z\cos\theta_0)\}$$

$$w^0(x,z)=-ik^*c_0^2\delta\cos\theta_0\exp\{ik^*(x\sin\theta_0-z\cos\theta_0)\}.$$

We have denoted

$$\delta=\frac{1+(\nu-\nu')i\omega^*/c_0^2}{i\omega^*-\nu k^{*2}}.$$

Then, we write

$$p(x,z)=\begin{cases}p^0(x,z)+p^+(x,z), & \text{in } \mathcal{D}^+\\ p^-(x,z), & \text{in } \mathcal{D}^-\end{cases},$$

$$u(x,z)=\begin{cases}u^0(x,z)+u^+(x,z), & \text{in } \mathcal{D}^+\\ u^-(x,z), & \text{in } \mathcal{D}^-\end{cases},$$

$$w(x,z)=\begin{cases}w^0(x,z)+w^+(x,z), & \text{in } \mathcal{D}^+\\ w^-(x,z), & \text{in } \mathcal{D}^-\end{cases}.$$

The functions $p^\pm(x,z)$ satisfy Eq. (11) in the corresponding domains $\mathcal{D}^\pm$ and the functions $u^\pm(x,z)$, $w^\pm(x,z)$ are solutions of Eq. (10). Since the array of scatterers is periodic over the ox axis, the pressure and velocity fields can be written as

$$p^\pm(x,z)=\exp\{ik^*x\sin\theta_0\}\tilde{p}^\pm(x,z), \tag{14}$$

$$\mathbf{v}^\pm(x,z)=\exp\{ik^*x\sin\theta_0\}\tilde{\mathbf{v}}^\pm(x,z), \tag{15}$$

where $\tilde{p}^\pm(x,z)$ and $\tilde{\mathbf{v}}^\pm(x,z)$ are periodic functions with respect to x

$$\tilde{p}^\pm(x+T,z)=\tilde{p}^\pm(x,z),$$

$$\tilde{\mathbf{v}}^\pm(x+T,z)=\tilde{\mathbf{v}}^\pm(x,z).$$

Equation (11), taking into consideration the periodicity of the function $\tilde{p}^{\pm}(x,z)$, gives

$$p^{\pm}(x,z)/\rho_0 = P_0^{\pm} \exp\{ik^*(x \sin\theta_0 \pm z \cos\theta_0)\} + \sum_{n\neq 0} P_n^{\pm} \exp(ik_n x)\exp(\mp r_n z), \qquad (16)$$

where

$$k_n = n\omega + k^* \sin\theta_0, \quad \omega = 2\pi/T,$$

$$r_n = \sqrt{k_n^2 - k^{*2}}, \quad \mathrm{Re}(r_n) > 0.$$

Thus, the scattered pressure field consists of a superposition of an infinite number of wave modes. For small frequences $[0 < k^* T/2 < \pi/(1-\sin\theta_0)]$ only the lowest-order mode is propagating; all the other modes describe waves which decay exponentially with the distance to the plane $z=0$. In writing the solution (16) we considered also the Sommerfeld radiation condition. The constants $P_n^{\pm}$ will be deteminded by using the boundary conditions. In the case of the viscous fluid, these conditions are written by means of velocities.

To determine the representation formulas for the velocity field we use Eq. (10), where we introduce the form (16) for the pressure. Taking into consideration also the abovementioned periodicity results in

$$u^{\pm}(x,z) = \sum_{n=-\infty}^{\infty} u_n^{\pm}(x,z),$$

$$w^{\pm}(x,z) = \sum_{n=-\infty}^{\infty} w_n^{\pm}(x,z),$$

where

$$u_0^{\pm}(x,z) = ik^* \sin\theta_0 \delta P_0^{\pm} \exp\{ik^*(x\sin\theta_0 \pm z\cos\theta_0)\} + \frac{H_0^{\pm}}{ik_0}\exp\{ik_0 x \mp q_0 z\},$$

$$w_0^{\pm}(x,z) = \pm ik^* \cos\theta_0 \delta P_0^{\pm} \exp\{ik^*(x\sin\theta_0 \pm z\cos\theta_0)\} \pm \frac{H_0^{\pm}}{q_0}\exp\{ik_0 x \mp q_0 z\}, \qquad (17)$$

$$u_n^{\pm}(x,z) = \frac{H_n^{\pm}}{ik_n}\exp\{ik_n x \mp q_n z\} + ik_n \delta P_n^{\pm} \exp\{ik_n x \mp r_n z\},$$

$$w_n^{\pm}(x,z) = \pm \frac{H_n^{\pm}}{q_n}\exp\{ik_n x \mp q_n z\} \mp r_n \delta P_n^{\pm} \exp\{ik_n x \mp r_n z\}. \qquad (18)$$

The constants $H_n^{\pm}$ have to be determined by the boundary conditions. We have also denoted

$$q_n = \sqrt{k_n^2 - i\omega^*/\nu}, \quad \mathrm{Re}(q_n) > 0.$$

We note that naturally the solution for velocity has been decomposed into two parts: a propagating mode including the P terms and a diffusive mode, driven by viscosity, including the H terms.

IV. THE HYPERSINGULAR INTEGRAL EQUATIONS OF THE PROBLEM

Let us consider the functions

$$\tilde{u}(x) \equiv \tilde{u}^-(x,0) = \sum_{n=-\infty}^{\infty} \tilde{u}_n \exp(in\omega x), \quad x \in \mathbb{R} \qquad (19)$$

$$\tilde{w}(x) \equiv \tilde{w}^-(x,0) = \sum_{n=-\infty}^{\infty} \tilde{w}_n \exp(in\omega x), \quad x \in \mathbb{R} \qquad (20)$$

where the Fourier coefficients are given by the formula

$$[\tilde{u}_n, \tilde{w}_n] = \frac{1}{T}\int_{-T/2}^{T/2} [\tilde{u}(x), \tilde{w}(x)] \exp(-in\omega x)dx. \qquad (21)$$

Since the velocity is vanishing on the rigid boundaries between the openings, the functions $\tilde{u}(x)$, $\tilde{w}(x)$ are different from zero only along the apertures. Due to periodicity, it is sufficient to determine them along the interval $(-a,a)$.

The continuity of the velocity across the apertures and screens yields also the relationships

$$u^0(x,0)\exp\{-ik^* x \sin\theta_0\} + \tilde{u}^+(x,0) = \tilde{u}(x), \qquad (22)$$

$$w^0(x,0)\exp\{-ik^* x \sin\theta_0\} + \tilde{w}^+(x,0) = \tilde{w}(x), \qquad (23)$$

valid along the whole axis. The relationships (19)–(23) allow us to determine the constants $P_n^{\pm}$ and $H_n^{\pm}$ in terms of the Fourier coefficients $\tilde{u}_n$ and $\tilde{w}_n$ as

$$\begin{aligned}
P_0^+ &= \frac{q_0\tilde{w}_0 - ik^*\tilde{u}_0 \sin\theta_0}{iq_0\cos\theta_0 + k^*\sin^2\theta_0}\frac{1}{\delta k^*} + \frac{iq_0\cos\theta_0 - k^*\sin^2\theta_0}{iq_0\cos\theta_0 + k^*\sin^2\theta_0},\\
P_0^- &= \frac{-q_0\tilde{w}_0 - ik^*\tilde{u}_0 \sin\theta_0}{iq_0\cos\theta_0 + k^*\sin^2\theta_0}\frac{1}{\delta k^*},\\
H_0^+ &= ik^*\tilde{u}_0\sin\theta_0 + \delta k^{*2}(1+P_0^+)\sin^2\theta_0,\\
H_0^- &= ik^*\tilde{u}_0\sin\theta_0 + \delta k^{*2}P_0^-\sin^2\theta_0,\\
P_n^{\pm} &= \frac{\pm q_n\tilde{w}_n - ik_n\tilde{u}_n}{\delta(k_n^2 - r_n q_n)}, \quad n\neq 0,\\
H_n^{\pm} &= \frac{\pm k_n^2 q_n \tilde{w}_n - ik_n r_n q_n \tilde{u}_n}{k_n^2 - r_n q_n}, \quad n \neq 0.
\end{aligned} \qquad (24)$$

To obtain the equations satisfied by the functions $\tilde{u}(x)$ and $\tilde{w}(x)$, we impose the condition of continuity of the normal derivative of velocity along the aperture

$$\frac{\partial u^0(x,0)}{\partial z}\exp\{-ik^* x\sin\theta_0\} + \frac{\partial \tilde{u}^+(x,0)}{\partial z} = \frac{\partial \tilde{u}^-(x,0)}{\partial z}, \quad x \in (-a,a), \qquad (25)$$

$$\frac{\partial w^0(x,0)}{\partial z}\exp\{-ik^* x\sin\theta_0\} + \frac{\partial \tilde{w}^+(x,0)}{\partial z} = \frac{\partial \tilde{w}^-(x,0)}{\partial z}, \quad x \in (-a,a). \qquad (26)$$

These "boundary relationships" will be associated with the boundary conditions along the screens

$$\tilde{u}(x)=0, \quad x\in(a,b),$$

$$\tilde{w}(x)=0, \quad x\in(a,b).$$

Taking the second primitive (indefinite integral) with respect to the x variable of the relationships (25) and (26) results in

$$\alpha_1\tilde{u}_0\frac{x^2}{2}+\sum_{n\neq 0}\frac{r_n}{k_n^2-r_nq_n}\frac{\tilde{u}_n}{(in\omega)^2}\exp\{in\omega x\}$$

$$=\frac{ad_1}{2T}x^2+c_1x+c_0, \quad x\in(-a,a), \tag{27}$$

$$\alpha_3\tilde{w}_0\frac{x^2}{2}+\sum_{n\neq 0}\frac{q_n}{k_n^2-r_nq_n}\frac{\tilde{w}_n}{(in\omega)^2}\exp\{in\omega x\}$$

$$=\frac{ad_3}{2T}x^2+c_1'x+c_0', \quad x\in(-a,a), \tag{28}$$

where c_1, c_0, c_1', c_0' are arbitrary integration constants and the constants α_1, α_3, d_1, d_3 are given by formulas

$$\alpha_1=\frac{-i\cos\theta_0}{iq_0\cos\theta_0+k^*\sin^2\theta_0},$$

$$\alpha_3=\frac{q_0/k^*}{iq_0\cos\theta_0+k^*\sin^2\theta_0},$$

$$d_1=\frac{c_0^2\delta k^*T\sin\theta_0\cos\theta_0/a}{iq_0\cos\theta_0+k^*\sin^2\theta_0},$$

$$d_3=\frac{-ic_0^2\delta q_0T\cos\theta_0/a}{iq_0\cos\theta_0+k^*\sin^2\theta_0}.$$

The asymptotic developments

$$\frac{r_n}{k_n^2-r_nq_n}=\frac{|n\omega|}{A}+\frac{k^*\sin\theta_0}{A}\frac{n}{|n|}+\frac{C_1}{A}\frac{1}{|n\omega|}+O\left(\frac{1}{|n\omega|^2}\right),$$

$$\frac{q_n}{k_n^2-r_nq_n}=\frac{|n\omega|}{A}+\frac{k^*\sin\theta_0}{A}\frac{n}{|n|}+\frac{C_3}{A}\frac{1}{|n\omega|}+O\left(\frac{1}{|n\omega|^2}\right),$$

where

$$A=\frac{1}{2}\left(k^{*2}+\frac{i\omega^*}{\nu}\right), \quad B=\frac{i\omega^*k^{*2}}{2A\nu},$$

$$C_1=B-\frac{3k^{*2}}{4}-\frac{i\omega^*}{4\nu}, \quad C_3=B-\frac{k^{*2}}{4}-\frac{3i\omega^*}{4\nu},$$

also prove true.

Since $\tilde{u}_n$ are the Fourier coefficients of a continuous function, we have also $\tilde{u}_n=o(n^{-1})$ and consequently

$$\left|\frac{r_n}{k_n^2-r_nq_n}\frac{\tilde{u}_n}{(in\omega x)^2}\right|<\frac{\text{const}}{n^2}.$$

Hence, the infinite series in formula (27) is converging uniformly for $x\in[-a,a]$. According to property (*f*) in Ref. 9 (or Ref. 10) the relationship (27) can be differentiated term by term any number of times and the formulas obtained this way are valid as distributions (generalized functions).

The second derivative of relationship (27) with respect to the x variable can be written as

$$\frac{1}{A}\frac{d^2}{dx^2}\sum_{n\neq 0}\frac{|n\omega|\tilde{u}_n}{(in\omega)^2}\exp\{in\omega x\}$$

$$+\frac{k^*\sin\theta_0}{A}\frac{d^2}{dx^2}\sum_{n\neq 0}\frac{n}{|n|}\frac{\tilde{u}_n}{(in\omega)^2}\exp\{in\omega x\}$$

$$+\frac{C_1}{A}\sum_{n\neq 0}\frac{\tilde{u}_n}{|n\omega|}\exp\{in\omega x\}+\alpha_1\tilde{u}_0$$

$$+\sum_{n\neq 0}\left[\frac{r_n}{k_n^2-r_nq_n}-\frac{|n\omega|}{A}-\frac{k^*\sin\theta_0}{A}\frac{n}{|n|}-\frac{C_1}{A}\frac{1}{|n\omega|}\right]\tilde{u}_n\exp\{in\omega x\}=\frac{ad_1}{T}, \quad x\in(-a,a). \tag{29}$$

Since the last infinite series in this relationship is uniformly convergent, its sum is a smooth function of x along the interval $(-a,a)$. Let us denote

$$K_1^R(x)=\alpha_1+\sum_{n\neq 0}\left[\frac{r_n}{k_n^2-r_nq_n}-\frac{|n\omega|}{A}-\frac{k^*\sin\theta_0}{A}\frac{n}{|n|}-\frac{C_1}{A}\frac{1}{|n\omega|}\right]\exp\{in\omega x\},$$

$$K_3^R(x)=\alpha_3+\sum_{n\neq 0}\left[\frac{q_n}{k_n^2-r_nq_n}-\frac{|n\omega|}{A}-\frac{k^*\sin\theta_0}{A}\frac{n}{|n|}-\frac{C_3}{A}\frac{1}{|n\omega|}\right]\exp\{in\omega x\}.$$

Introducing the expression of the Fourier coefficients, we obtain

$$\mathbb{I}_R(x)\equiv\alpha_1\tilde{u}_0+\sum_{n\neq 0}\left[\frac{r_n}{k_n^2-r_nq_n}-\frac{|n\omega|}{A}-\frac{k^*\sin\theta_0}{A}\frac{n}{|n|}-\frac{C_1}{A}\frac{1}{|n\omega|}\right]\tilde{u}_n\exp\{in\omega x\}$$

$$=\frac{1}{T}\int_{-a}^{a}\tilde{u}(x')K_1^R(x-x')dx'.$$

The other infinite series will give the singular terms of Eq. (29). We analyze now each of the singular terms. Thus, substituting again the Fourier coefficients results in

$$\mathbb{I}_3 \equiv \sum_{n\neq 0} \frac{\tilde{u}_n}{|n\omega|} \exp\{in\omega x\}$$

$$= \sum_{n\neq 0} \frac{1}{T} \int_{-a}^{+a} \tilde{u}(x') \frac{\exp\{in\omega(x-x')\}}{|n\omega|} dx'$$

$$= \frac{1}{\pi} \int_{-a}^{+a} \tilde{u}(x') \sum_{n=1}^{\infty} \frac{\cos[n\omega(x-x')]}{n} dx'$$

$$= -\frac{1}{\pi} \int_{-a}^{+a} \tilde{u}(x') \log\left| 2 \sin \frac{\omega(x-x')}{2} \right| dx'. \tag{30}$$

Here, the formula

$$\sum_{n=1}^{\infty} \frac{\cos(n\omega x)}{n} = -\log\left| 2 \sin \frac{\omega x}{2} \right|,$$

given in Ref. 11 (formula 1.442) or Ref. 12 has been used. Therefore

$$\mathbb{I}_3 = -\frac{1}{\pi} \int_{-a}^{+a} \tilde{u}(x') \log\left| \left[\sin \frac{\omega(x-x')}{2} \right] \left[\frac{\omega(x-x')}{2} \right]^{-1} \right| dx'$$

$$-\frac{1}{\pi} \int_{-a}^{+a} \tilde{u}(x') \log|\omega(x-x')| dx'. \tag{31}$$

The first integral in this relationship is regular and the second one has a weak (logarithmic) singularity for $x' = x$.

Similarly, the second singular term in Eq. (29) becomes

$$\mathbb{I}_2 \equiv \frac{d^2}{dx^2} \sum_{n\neq 0} \frac{n}{|n|} \frac{\tilde{u}_n}{(in\omega)^2} \exp\{in\omega x\}$$

$$= -\frac{d}{dx} \frac{i}{\pi} \int_{-a}^{+a} \tilde{u}(x') \sum_{n=1}^{\infty} \frac{\cos[n\omega(x-x')]}{n} dx'.$$

Hence

$$\mathbb{I}_2(x) = i \frac{d\mathbb{I}_3}{dx} = \frac{i}{\pi} \int_{-a}^{+a} \tilde{u}(x') \left[\frac{\omega}{2} \cot \frac{\omega(x-x')}{2} - \frac{1}{x-x'} \right] dx'$$

$$+ \frac{i}{\pi} \frac{d}{dx} \int_{-a}^{+a} \tilde{u}(x') \log|\omega(x-x')| dx'. \tag{32}$$

Again, the first integral in formula (32) is regular. The second one can be written as

$$\frac{i}{\pi} \frac{d}{dx} \int_{-a}^{+a} \tilde{u}(x') \log|\omega(x-x')| dx' = \frac{i}{\pi} \int_{-a}^{+a} \tilde{u}(x') \frac{dx'}{x-x'}, \tag{33}$$

and represents a finite part (or principal value) integral (see Refs. 13, 9, and 14). Finally, the second term in Eq. (29) has the form

$$\mathbb{I}_2(x) = \frac{i}{\pi} \int_{-a}^{+a} \tilde{u}(x') \left[\frac{\omega}{2} \cot \frac{\omega(x-x')}{2} - \frac{1}{x-x'} \right] dx'$$

$$+ \frac{i}{\pi} \int_{-a}^{+a} \tilde{u}(x') \frac{dx'}{x-x'}. \tag{34}$$

For the first singular term in (29), performing similar transformations results in

$$\mathbb{I}_1 \equiv \frac{d^2}{dx^2} \sum_{n\neq 0} \frac{|n\omega| \tilde{u}_n}{(in\omega)^2} \exp\{in\omega x\} = -\frac{d^2 \mathbb{I}_3(x)}{dx^2}$$

$$= \frac{d^2}{dx^2} \frac{1}{\pi} \int_{-a}^{+a} \tilde{u}(x') \log|\omega(x-x')| dx'$$

$$+ \frac{d^2}{dx^2} \frac{1}{\pi} \int_{-a}^{+a} \tilde{u}(x') \log\left| \left[\sin \frac{\omega(x-x')}{2} \right] \right.$$

$$\left. \times \left[\frac{\omega(x-x')}{2} \right]^{-1} \right| dx'.$$

The last integral in this formula is regular and the other one can be written as a finite part. Finally, the first singular term becomes

$$\mathbb{I}_1 = -\frac{1}{\pi} \int_{-a}^{+a} \tilde{u}(x') \left[\frac{\omega^2}{4} \sin^{-2} \frac{\omega(x-x')}{2} \right.$$

$$\left. -\frac{1}{(x-x')^2} \right] dx' - \frac{1}{\pi} \int_{-a}^{+a} \tilde{u}(x') \frac{dx'}{(x-x')^2}.$$

In this formula

$$\int_{-a}^{+a} \tilde{u}(x') \frac{dx'}{(x-x')^2} = -\frac{d}{dx} \int_{-a}^{+a} \tilde{u}(x') \frac{dx'}{x-x'}$$

$$= -\frac{d^2}{dx^2} \int_{-a}^{+a} \tilde{u}(x') \log|\omega(x-x')| dx, \tag{35}$$

is a finite part integral (see Refs. 13, 15, and 14).

Bringing together all the above transformations, Eq. (29) becomes

$$-\frac{1}{\pi A} \int_{-a}^{+a} \tilde{u}(x') \frac{dx'}{(x-x')^2} + \frac{ik^* \sin\theta_0}{\pi A} \int_{-a}^{+a} \tilde{u}(x') \frac{dx'}{x-x'}$$

$$-\frac{C_1}{\pi A} \int_{-a}^{+a} \tilde{u}(x') \log|\omega(x-x')| dx'$$

$$+\frac{1}{T} \int_{-a}^{a} \tilde{u}(x') K_1^R(x-x') dx' - \frac{1}{\pi A} \int_{-a}^{+a} \tilde{u}(x')$$

$$\times \left[\frac{\omega^2}{4} \sin^{-2} \frac{\omega(x-x')}{2} - \frac{1}{(x-x')^2} \right] dx'$$

$$+\frac{ik^* \sin\theta_0}{\pi A} \int_{-a}^{+a} \tilde{u}(x') \left[\frac{\omega}{2} \cot \frac{\omega(x-x')}{2} - \frac{1}{x-x'} \right] dx'$$

$$-\frac{C_1}{\pi A} \int_{-a}^{+a} \tilde{u}(x') \log\left| \left[\sin \frac{\omega(x-x')}{2} \right] \left[\frac{\omega(x-x')}{2} \right]^{-1} \right| dx'$$

$$= \frac{a d_1}{T}, \quad x \in (-a, a). \tag{36}$$

Performing the same transformations with Eq. (28), a similar equation results for the function $\widetilde{w}(x)$

$$-\frac{1}{\pi A}\int_{-a}^{+a}\widetilde{w}(x')\frac{dx'}{(x-x')^2}+\frac{ik^*\sin\theta_0}{\pi A}\int_{-a}^{+a}\widetilde{w}(x')\frac{dx'}{x-x'}$$
$$-\frac{C_3}{\pi A}\int_{-a}^{+a}\widetilde{w}(x')\log|\omega(x-x')|dx'$$
$$+\frac{1}{T}\int_{-a}^{a}\widetilde{w}(x')K_3^R(x-x')dx'-\frac{1}{\pi A}\int_{-a}^{+a}\widetilde{w}(x')$$
$$\times\left[\frac{\omega^2}{4}\sin^{-2}\frac{\omega(x-x')}{2}-\frac{1}{(x-x')^2}\right]dx'$$
$$+\frac{ik^*\sin\theta_0}{\pi A}\int_{-a}^{+a}\widetilde{w}(x')\left[\frac{\omega}{2}\cot\frac{\omega(x-x')}{2}-\frac{1}{x-x'}\right]dx'$$
$$-\frac{C_3}{\pi A}\int_{-a}^{+a}\widetilde{w}(x')\log\left|\left[\sin\frac{\omega(x-x')}{2}\right]\left[\frac{\omega(x-x')}{2}\right]^{-1}\right|dx'$$
$$=\frac{ad_3}{T},\quad x\in(-a,a). \tag{37}$$

Equations (36) and (37) will be solved in Sec. V by using a Galerkin-type method.

Since there are numerical methods tailored specially for finite-part integrals, we shall write also a different form of these equations. So, by using the finite-part definition for more general functions[13] and Ref. 16, pp. 64–66, problem 5, Eqs. (36) and (37) can be written as

$$-\frac{\omega}{2A}\int_{-a}^{a}\frac{\widetilde{u}(x')dx'}{\sin^2(\omega(x-x')/2)}$$
$$+\frac{ik^*\sin\theta_0}{A}\int_{-a}^{a}\widetilde{u}(x')\cot\frac{\omega(x-x')}{2}dx'$$
$$-\frac{2C_1}{\omega A}\int_{-a}^{a}\widetilde{u}(x')\log\left|2\sin\frac{\omega(x-x')}{2}\right|dx'$$
$$+\int_{-a}^{a}\widetilde{u}(x')K_1^R(x-x')dx'=ad_1 \tag{38}$$

$$-\frac{\omega}{2A}\int_{-a}^{a}\frac{\widetilde{w}(x')dx'}{\sin^2(\omega(x-x')/2)}$$
$$+\frac{ik^*\sin\theta_0}{A}\int_{-a}^{a}\widetilde{w}(x')\cot\frac{\omega(x-x')}{2}dx'$$
$$-\frac{2C_3}{\omega A}\int_{-a}^{a}\widetilde{w}(x')\log\left|2\sin\frac{\omega(x-x')}{2}\right|dx'$$
$$+\int_{-a}^{a}\widetilde{w}(x')K_3^R(x-x')dx'=ad_3. \tag{39}$$

Notice that both integral equations have the same singular part. As the solutions are satisfying the conditions $\widetilde{u}(\pm a)=\widetilde{w}(\pm a)=0$ the equations can be transformed into integro-differential equations with Cauchy-type singularities. Numerical methods for solving such equations were developed by Multhopp-Kalandiya,[17] Kutt,[18] and Dragos.[19]

Remark 1. *These types of singular integral equations involving second-order "poles" arise naturally in various physical problems.*[14,15,20,21] *They are called "integral equations with strong singularities" in some papers and "hypersingular integral equations" in other papers. Since in the field of wave propagation and acoustics the last denomination is used more often, we adopted it also in the present work.*

Finally, we note that using a Rayleigh-type representation of velocity field, by means of a potential function $\phi(x,z)$ and a stream function $\psi(x,z)$ the solution of the viscous diffraction problem reduces to the same hypersingular integral equations (38) and (39). Hence, these equations characterize the viscous diffraction problem by a grating and are not the result of a particular approach.

V. REDUCTION OF THE HYPERSINGULAR INTEGRAL EQUATIONS TO INFINITE SYSTEMS OF ALGEBRAIC EQUATIONS

Instead of using collocation methods for approximating the solution of the singular integral equations (38) and (39), we prefer a Galerkin-type approach based on a special basis of the corresponding Hilbert space. The method takes advantage of the form of integral equations by using some spectral-type relationships, fast Fourier transform of some smooth function, and summation of rapid convergent infinite series.

A. Galerkin's method for solving the integral equations

In the space $\mathcal{H}^{1/2}(-a,a)$ of functions continuous on $[-a,a]$ with derivatives having singularities of order 1/2 at extremities, we consider the basis

$$\left\{\sin\left(n\arccos\frac{x}{a}\right)\right\},\quad n=1,2,\ldots.$$

We represent the function $\widetilde{u}(x)$ as

$$\widetilde{u}(x)=\sum_{m=1}^{\infty}u_m\sin\left(m\arccos\frac{x}{a}\right).$$

Equation (36) becomes

$$\sum_{m=1}^{\infty} u_m \left\{ -\frac{2C_1}{\omega A} \int_{-a}^{+a} \sin(m\theta') \log|\omega(x-x')| dx' + \frac{2ik^* \sin\theta_0}{\omega A} \int_{-a}^{+a} \frac{\sin(m\theta') dx'}{x-x'} \frac{-2}{\omega A} \int_{-a}^{+a} \frac{\sin(m\theta') dx'}{(x-x')^2} - \frac{2}{\omega A} \int_{-a}^{+a} \sin(m\theta') \left[\frac{\omega^2}{4} \sin^{-2} \frac{\omega(x-x')}{2} - \frac{1}{(x-x')^2} \right] dx' + \frac{2ik^* \sin\theta_0}{\omega A} \int_{-a}^{+a} \sin(m\theta') \times \left[\frac{\omega}{2} \cot \frac{\omega(x-x')}{2} - \frac{1}{x-x'} \right] dx' + \int_{-a}^{a} \sin(m\theta') K_1^R(x-x') dx' - \frac{2C_1}{\omega A} \int_{-a}^{+a} \sin(m\theta') \log \left| \left[\sin \frac{\omega(x-x')}{2} \right] \times \left[\frac{\omega(x-x')}{2} \right]^{-1} \right| dx' = a d_1, \quad x \in (-a,a),$$

where we have denoted $\theta' = \arccos(x'/a)$.

Applying the *Galerkin's method* results in

$$\sum_{m=1}^{\infty} [S_{pm}^0 + C_1 \hat{S}_{pm}^0] u_m + \sum_{m=1}^{\infty} [S_{pm}^R + C_1 \hat{S}_{pm}^R + R_{pm}^{(1)}] u_m = \frac{d_1}{2\pi} \delta_{p,1}, \quad p = 1,2,\ldots.$$

The singular terms are

$$S_{pm}^0 = \frac{-2}{\omega a^2 A} \frac{1}{\pi^2} \int_0^{\pi} \int_0^{\pi} \frac{dS}{(\cos\theta - \cos\theta')^2} + \frac{2ik^* \sin\theta_0}{\omega a A} \frac{1}{\pi^2} \int_0^{\pi} \int_0^{\pi} \frac{dS}{\cos\theta - \cos\theta'},$$

$$\hat{S}_{pm}^0 = -\frac{2}{\omega A} \frac{1}{\pi^2} \int_0^{\pi} \int_0^{\pi} \log|\cos\theta - \cos\theta'| dS,$$

and the regular part contains the terms

$$S_{pm}^R = \frac{1}{\pi^2} \int_0^{\pi} \int_0^{\pi} \left\{ -\frac{\omega}{2A} \left[\frac{1}{\sin^2 Z} - \frac{1}{Z^2} \right] + \frac{ik^* \sin\theta_0}{A} \left[\cot Z - \frac{1}{Z} \right] \right\} dS, \tag{40}$$

$$\hat{S}_{pm}^R = -\frac{2}{\omega A} \frac{1}{\pi^2} \int_0^{\pi} \int_0^{\pi} \log \left| \frac{\sin Z}{Z} \right| dS, \tag{41}$$

$$R_{pm}^{(1)} = \frac{1}{a^2 \pi^2} \int_{-a}^{a} \int_{-a}^{a} \sin(m\theta') \sin(p\theta) K_1^R(x-x') dx' \, dx.$$

We have denoted $\theta = \arccos(x/a)$, and also

$$dS = \sin(m\theta') \sin(p\theta) \sin\theta' \sin\theta \, d\theta \, d\theta',$$

$$Z = \frac{\omega a}{2} (\cos\theta - \cos\theta').$$

Using formulas given in the Appendix results in

$$S_{pm}^0 = \frac{m}{\omega a^2 A} \delta_{p,m} + \frac{ik^* \sin\theta_0}{2\omega a A} (\delta_{p,m+1} - \delta_{p+1,m}) \tag{42}$$

$$\hat{S}_{pm}^0 = \frac{1}{4\omega A} \begin{cases} (\delta_{p,m} - \delta_{p,m+2})/(m+1) \\ \quad + (\delta_{p,m} - \delta_{p+2,m})/(m-1), & m \neq 1, \\ (\delta_{p,1} - \delta_{p,3})/2 - 2\log(\omega a/2) \delta_{p,1}, & m = 1 \end{cases} \tag{43}$$

$$R_{pm}^{(1)} = \frac{\alpha_1}{4} \delta_{p,1} \delta_{m,1} + i^{p-m} m p \sum_{n=1}^{\infty} \frac{J_p(an\omega) J_m(an\omega)}{(an\omega)^2} \times \left[\frac{r_n}{k_n^2 - r_n q_n} + (-1)^{p+m} \frac{r_{-n}}{k_{-n}^2 - r_{-n} q_{-n}} - \frac{1 + (-1)^{p+m}}{A} \left(n\omega + \frac{C_1}{n\omega} \right) - \frac{1 - (-1)^{p+m}}{A} k^* \sin\theta_0 \right]. \tag{44}$$

Finally, we obtain the infinite system of algebraic equations for determining the coefficients u_n

$$\sum_{m=1}^{\infty} [S_{pm}^0 + C_1 \hat{S}_{pm}^0 + S_{pm}^R + C_1 \hat{S}_{pm}^R + R_{pm}^{(1)}] u_m = \frac{d_1}{2\pi} \delta_{p,1}, \quad p = 1,2,\ldots. \tag{45}$$

Similarly, we write

$$\tilde{w}(x) = \sum_{m=1}^{\infty} w_m \sin\left(m \arccos \frac{x}{a} \right),$$

and the integral equation (39) yields the infinite system of algebraic equations for the coefficients w_m

$$\sum_{m=1}^{\infty} [S_{pm}^0 + C_3 \hat{S}_{pm}^0 + S_{pm}^R + C_3 \hat{S}_{pm}^R + R_{pm}^{(3)}] u_m = \frac{d_3}{2\pi} \delta_{p,1}, \quad p = 1,2,\ldots. \tag{46}$$

The coefficients $R_{pm}^{(3)}$ have the expression

$$R_{pm}^{(3)} = \frac{\alpha_3}{4} \delta_{p,1} \delta_{m,1} + i^{p-m} m p \sum_{n=1}^{\infty} \frac{J_p(an\omega) J_m(an\omega)}{(an\omega)^2} \times \left[\frac{q_n}{k_n^2 - r_n q_n} + (-1)^{p+m} \frac{q_{-n}}{k_{-n}^2 - r_{-n} q_{-n}} - \frac{1 + (-1)^{p+m}}{A} \left(n\omega + \frac{C_3}{n\omega} \right) - \frac{1 - (-1)^{p+m}}{A} k^* \sin\theta_0 \right]. \tag{47}$$

B. Numerical realization of the method

The systems of linear equations (45) and (46) have good properties from a computational point of view. Thus, the coefficients S_{pm}^{0}, $\hat{S}_{pm}^{0}$ are given by explicit relationships (42) and (43). The coefficients S_{pm}^{R}, $\hat{S}_{pm}^{R}$ are expressed by 2D cosine Fourier transforms. The integrands in formulas (40) and (41) are smooth functions such that these integrals can be computed efficiently by using the 2D discrete cosine transform function of MATLAB. Finally, the coefficients $R_{pm}^{(1)}$, $R_{pm}^{(3)}$ can be obtained directly by summing the infinite series in formulas (44) and (47). By subtracting several terms, and summing them separately, we have transformed the initial infinite series into rapid convergent series.

In fact, by the approch used in the previous section, we have turned the convolution operators in Fourier transform domain; this transform converts the convolution to a product ‘‘quasidiagonalizing’’ the operators. Thus, the strongly singular operator contributes only on the main diagonal of the system, the principal value integral has contributions on the two diagonals next to the principal one, and the term containing the logarithmic singularity contributes in the system to coefficients lying on the principal diagonal and the four closer to it. This is why the finite sections of the resulting infinite systems of linear equations have relatively low condition numbers and the series giving the functions $\tilde{u}(x)$, $\tilde{w}(x)$ are rapidly convergent.

VI. SOUND TRANSMISSION FACTOR. NUMERICAL RESULTS

The important element in this analysis is the modulus t of the *sound transmission factor*,[22] $t=|P_0^-/c_0^2|$, which shows ‘‘how much’’ of the incoming plane wave is passing through the grating. Thus, once the solutions of the systems (45) and (46) are determined the functions $\tilde{u}(x)$, $\tilde{w}(x)$ can be introduced in formula (21) to obtain the corresponding Fourier coefficients. Finally, the formula (24) provides the value of the modulus of sound transmission factor as

$$t\equiv|P_0^-/c_0^2|=\left|\frac{q_0\tilde{w}_0+ik^*\tilde{u}_0\sin\theta_0}{iq_0\cos\theta_0+k^*\sin^2\theta_0}\,\frac{a\pi}{2(a+b)c_0^2\delta k^*}\right|. \tag{48}$$

In Fig. 2(a) we have plotted the modulus t of sound transmission factor versus slit width d for various values of the periodic spacing w by including the influence of viscosity. All the calculations assumed a value of $\pi/4$ for the angle θ_0. In Fig. 2(b) the same problem was solved for the case where the viscosity of the air is neglected. It is very clear that, especially for very narrow slits, the viscosity of the air causes significant attenuation of the transmission wave. The results of Fig. 2(b) agree very well (the first three digits are the same) with those obtained by using the formulas provided in Refs. 3 and 4. Figure 3 shows again the dependence of modulus t of sound transmission factor upon d for different values of the incidence angle θ_0.

These results indicate that, in a simplified model of the screen, treated as a periodic array of slits in a substrate, the openings in the screen should be on the order of at least 10 μm in order to avoid excessive attenuation of the signal.

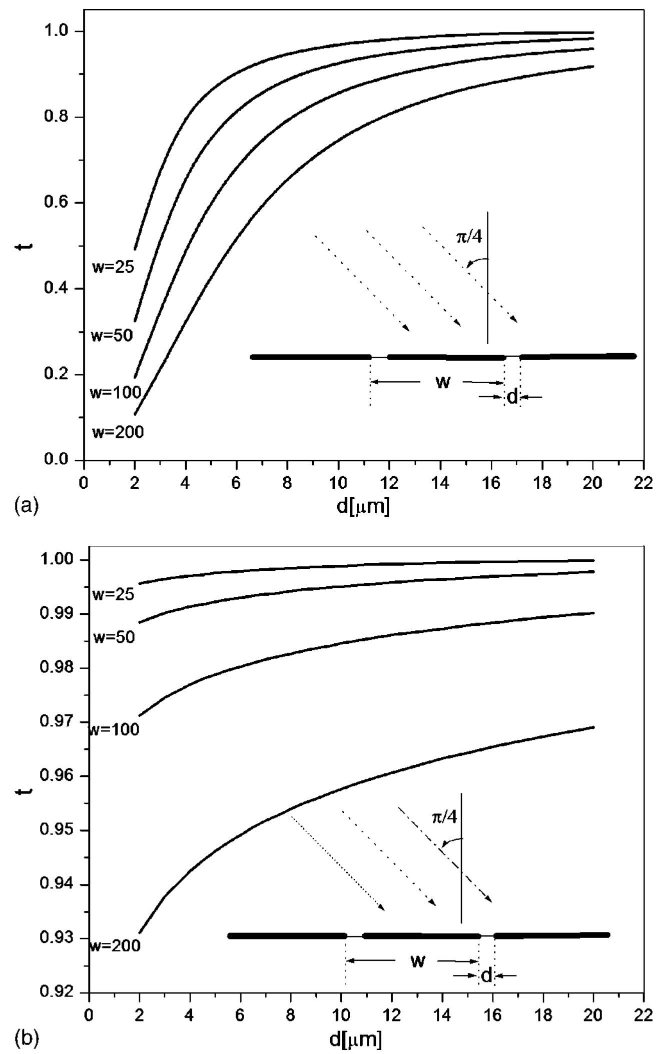

FIG. 2. (a) Modulus t of sound transmission factor versus slits' width for periodic spacing $w=25$, 50, 100, and 200 μm, $f=20$ KHz, and incidence angle $\theta_0=\pi/4$. The viscous case. (b) Modulus t of sound transmission factor versus slits' width for periodic spacing $w=25$, 50, 100, and 200 μm, $f=20$ KHz, and incidence angle $\theta_0=\pi/4$. The nonviscous case.

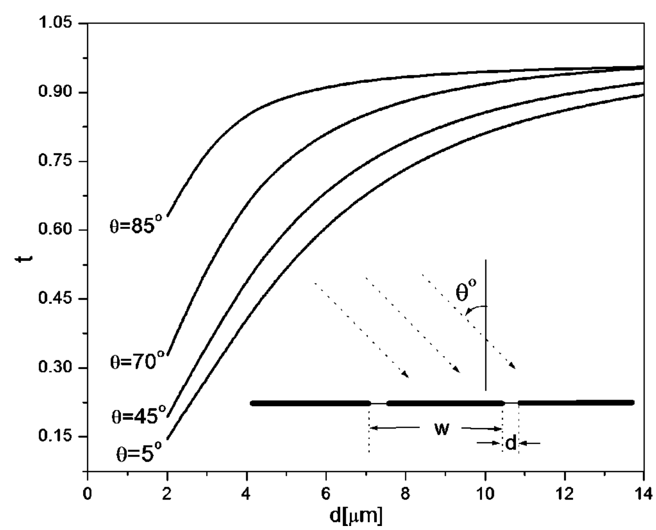

FIG. 3. Modulus t of sound transmission factor versus slits' width d for $w=100$ μm, $f=20$ KHz, and incidence angles $\theta_0=5$, 45, 70, and 85 deg.

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APPENDIX: SOME DEFINITE INTEGRALS

In the following, we evaluate some integrals used in Sec. IV. Starting with the formula[23,24]

$$-\frac{1}{\pi}\int_{-1}^{1}\frac{\cos(n\arccos x')}{\sqrt{1-x'^2}}\log|x-x'|dx' =\begin{cases}\cos(n\arccos x)/n, & n\geqslant 1\\ \log 2, & n=0\end{cases}, \tag{A1}$$

we can write also

$$-\frac{1}{\pi}\int_{-1}^{1}\sin(n\arccos x')\log|x-x'|dx' =\begin{cases}\dfrac{\cos(n-1)\theta}{2(n-1)}-\dfrac{\cos(n+1)\theta}{2(n+1)}, & n>1\\ \dfrac{\log 2}{2}-\dfrac{\cos(2\theta)}{4}, & n=1\end{cases}, \tag{A2}$$

where $\theta=\arccos x$. The derivatives of relation (A2), taking into consideration the definitions (33) and (35), give

$$\frac{1}{\pi}\int_{-1}^{1}\frac{\sin(n\arccos x')}{x-x'}dx'=\cos(n\theta),\quad n\geqslant 1, \tag{A3}$$

$$-\frac{1}{\pi}\int_{-1}^{1}\frac{\sin(n\arccos x')}{(x-x')^2}dx'=n\frac{\sin(n\theta)}{\sin\theta}. \tag{A4}$$

The change of integration variable $x'=\cos\theta'$ in the formulas (A2), (A3), and (A4) gives

$$-\frac{1}{\pi}\int_{0}^{\pi}\sin(n\theta')\sin\theta'\log|\cos\theta-\cos\theta'|d\theta' =\begin{cases}\dfrac{\cos(n-1)\theta}{2(n-1)}-\dfrac{\cos(n+1)\theta}{2(n+1)}, & n>1\\ \dfrac{\log 2}{2}-\dfrac{\cos(2\theta)}{4}, & n=1\end{cases}, \tag{A5}$$

$$\frac{1}{\pi}\int_{0}^{\pi}\frac{\sin(n\theta')\sin\theta'}{\cos\theta-\cos\theta'}d\theta'=\cos(n\theta),\quad n\geqslant 1, \tag{A6}$$

$$-\frac{1}{\pi}\int_{0}^{\pi}\frac{\sin(n\theta')\sin\theta'}{(\cos\theta-\cos\theta')^2}d\theta'=n\frac{\sin(n\theta)}{\sin\theta},\quad n\geqslant 1, \tag{A7}$$

the last two integrals being considered singular (finite-part) integrals.

For the last integral we consider the generating function of Bessel's functions

$$\exp\left\{\frac{z}{2}\left(w-\frac{1}{w}\right)\right\}=\sum_{n=-\infty}^{\infty}J_n(z)w^n,$$

and take $w=i\exp\{i\varphi\}$. This results in

$$\exp\{iz\cos\varphi\}=\sum_{n=-\infty}^{\infty}J_n(z)i^n\exp\{in\varphi\}.$$

Using the orthogonality relationship of the complex exponentials results in

$$\int_0^{\pi}\exp\{iz\cos\varphi\}\cos(m\varphi)d\varphi=\pi i^m J_m(z). \tag{A8}$$

Then, we have

$$\int_{-a}^{a}\sin\left(p\arccos\frac{x}{a}\right)\exp\{in\omega x\}dx =a\int_0^{\pi}\sin(p\theta)\sin\theta\exp\{in\omega\cos\theta\}d\theta,$$

and, using Eq. (A8) and some properties of Bessel's functions, we finally obtain the formula

$$\int_{-a}^{a}\sin\left(p\arccos\frac{x}{a}\right)\exp\{in\omega x\}dx=\pi i^{p-1}\frac{p}{n\omega}J_p(an\omega). \tag{A9}$$

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